Name:	Department: Comp Eng	GRADE
Student No:	Calculus 2 Final Exam	
Signature:	Date: 23/05/2017	

1. The series $\sum_{n=1}^{\infty} \frac{(-1)^n 4n^3}{n^4+1}$ converge absolutely $\boxed{F}$, converge conditionally $\boxed{T}$, diverges $\boxed{F}$. Give reason for your answer. (Write True(T) or False(F) in blanks.)

Absolute Convergence. $n^4+1 \leq n^4+n^4 = 2n^4$ for all $n \geq 1$.

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^n 4n^3}{n^4+1} \right| = \sum_{1}^{\infty} \frac{4n^3}{n^4+1} \geq \sum_{1}^{\infty} \frac{4n^3}{2n^4} = 2\sum_{n=1}^{\infty} \frac{1}{n} = +\infty$$ (since Harmonic series diverge to infinity)

So the series does not converge absolutely.

Conditional Convergence $a_n = \frac{4n^3}{n^4+1}$, $n \geq 1$.

i) $a_n \geq 0$ ✓ ii) $\lim_{n\to\infty} a_n = 0$ ✓

iii) a_n is decreasing for all $n \geq 2$. To see, let $f(n) = a_n = \frac{4n^3}{n^4+1}$

$$f'(n) = \frac{12n^2(n^4+1) - 4n^3\, 4n^3}{(n^4+1)^2} = \frac{-4n^6+12n^2}{(n^4+1)^2} = \frac{4n^2}{(n^4+1)^2}(3-n^4) < 0, \text{ for all } n \geq 2.$$

So $f(n) = a_n$ is decreasing for all $n \geq 2$.

By alternating series test, the series converge. (conditionally)

2. A unit vector parallel to the plane $2x-y-z=4$ and orthogonal to $i+j+k$ is $\boxed{-\frac{\sqrt{2}}{2}j + \frac{\sqrt{2}}{2}k}$

If $\vec{u}$ is parallel to the plane, $\vec{u}$ is orthogonal to the normal of the plane: $\vec{n} = 2i - j - k$.

So $\vec{u}$ is orthogonal to both $\vec{n}$ and $\vec{w} = i+j+k$. So $\vec{u} = \vec{n} \times \vec{w}$.

$$\vec{u} = \begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = i(-1+1) - j(2+1) + k(2+1) = -3j + 3k, \quad \frac{\vec{u}}{\|\vec{u}\|} = \frac{-3j+3k}{(3^2+3^2)^{1/2}}$$

3. Let $f(x,y) = g(x^2 y)$ where g is any differentiable function. $x\frac{\partial f}{\partial x} - 2y\frac{\partial f}{\partial y} = \boxed{0}$.

$$\frac{\partial f}{\partial x} = 2xy\, g'(x^2 y), \quad \frac{\partial f}{\partial y} = x^2 g'(x^2 y)$$

$$x\frac{\partial f}{\partial x} - 2y\frac{\partial f}{\partial y} = 2x^2 y\, g'(x^2 y) - 2x^2 y\, g'(x^2 y) = 0$$

4. The points on the surface $xy+yz+zx-x-z^2=0$ where the tangent plane is parallel to the xy-plane are $\boxed{(0,1,0)}$ and $\boxed{(-1/2, 1/2, 1/2)}$.

Let $F(x,y,z) = xy + yz + zx - x - z^2 = 0$, ∇F is normal to the surface $F(x,y,z)=0$.

$\nabla F = (y+z-1)\,i + (x+z)\,j + (y+x-2z)\,k$.

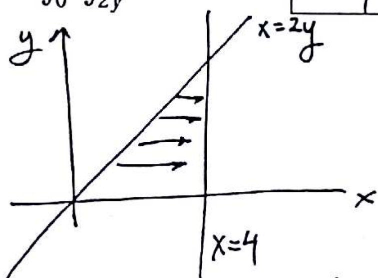
∇F is normal to the xy-plane if $y+z-1=0$, $x+z=0$.

$x=-z$, $y=1-z$.

$F(-z, 1-z, z) = 0 \Rightarrow z(1-2z)=0 \Rightarrow z=0$ or $z=1/2$.

$z=0 \Rightarrow x=0, y=1, \nabla F = k$, $\quad z=\frac{1}{2} \Rightarrow x=-\frac{1}{2}, y=\frac{1}{2}, \nabla F=-k$.

5. $\int_0^2 \int_{2y}^4 e^{-x^2}\,dx\,dy = \boxed{\dfrac{1-e^{-16}}{4}}$ (Hint: Reverse the order of integration)



$0 \le x \le 4$, $0 \le y \le x/2$.

$$\int_0^2 \int_{2y}^4 e^{-x^2}\,dx\,dy = \int_0^4 \int_0^{x/2} e^{-x^2}\,dy\,dx = \int_0^4 \frac{x}{2} e^{-x^2}\,dx$$

$(u=-x^2, \; du=-2x\,dx)$

$$= \frac{1}{2}\int_0^{-16} e^u \left(-\frac{1}{2}\right)du = -\frac{1}{4} e^u \Big|_0^{-16} = \frac{1-e^{-16}}{4}$$

6. Let R be the region bounded by the lines $x+y=0$, $x+y=1$, $2x-y=0$, $2x-y=3$. By making the change of variables $u=x+y$ and $v=2x-y$, the integral $\iint_R (x+y)^2\,dA$ is $\boxed{1/9}$.

Solving for $x, y \Rightarrow x = \frac{u+v}{3}$, $y = \frac{2u-v}{3}$.

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix} = \begin{vmatrix} 1/3 & 1/3 \\ 2/3 & -1/3 \end{vmatrix} = -\frac{1}{3}.$$

$$\iint_R (x+y)^2\,dA = \int_{u=0}^1 \int_{v=0}^1 u^2 \left|\frac{-1}{3}\right| dv\,du = \frac{1}{3}\int_0^1 u^2\,du = \frac{1}{9}.$$